Model for the coexistence of *d*-wave superconducting and charge-density-wave order parameters in high-temperature cuprate superconductors

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A theory describing a phase, where *d*-wave superconductivity coexists with charge-density waves (CDWs), has been developed. Numerical calculations were carried out for a specific case of cuprates when the CDW gap emerges at the nested antinodal regions of the two-dimensional Fermi surface. Different symmetries of the order parameters leads to their involved interplay so that the CDW one $\Sigma(T)$ is predicted to become temperature re-entrant for certain ranges of model parameters. Here, T is the temperature. At the same time, the superconducting energy gap $\Delta(T)$ deviates substantially from the canonical $d_{x^2-y^2}$ form. The CDW influence on $\Delta(T)$ is different from that on the critical temperature T_c. Hence, the resulting values of $2\Delta(0)/T_c$ are shown to fall into the range $5\div 8$, which is well known for high-T_c oxides and has not been explained yet.

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I. INTRODUCTION

The coexistence between superconductivity and charge-density waves (CDWs) in a large number of materials belongs to the well-known problems, intensively studied both experimentally and theoretically.[1–10] Nevertheless, a good many aspects remain unclear here due to an involved many-body character of the phenomenon. The problem became even more significant when the pseudogap phenomena in high-T_c oxides turned out to be by-products of the CDW normal-state background.[4–6,11–14] The pseudogap manifests itself in cuprates as a density-of-state (DOS) depletion both below and above the superconducting critical temperature T_c, a low-temperature (low-T) dip-hump structure in photoemission or tunnel spectra, or various charge modulation structures detected in scanning-tunnel microscopy (STM),[15] x-ray,[16,17] and neutron-scattering[18,19] measurements. Sometimes, those phenomena are blurred by intrinsic and extrinsic spatial inhomogeneities of oxide samples.[5,6,20] All those issues were studied by us in detail for *s*-wave superconductors.[5,6,21] Nevertheless, some of our results obtained earlier can be applicable to cuprates with certain reservations because the superconducting order parameter Δ in those materials is usually considered as possessing $d_{x^2-y^2}$ symmetry.[22] In this paper, we present a theory dealing with the coexistence of *d*-wave superconductivity and *partial* CDW gapping. Among other results, it allows, in particular, a puzzling phenomenon of an anomalously large ratio between the gap $\Delta(T=0)$ and T_c in cuprates[23–28] to be explained.

II. THEORY

The theory developed here is an extension of the Bilbro-McMillan approach, elaborated for *s*-wave Bardeen-Cooper-Schrieffer (BCS) superconductivity coexisting with CDWs or spin-density waves (SDWs),[4,21,29–40] to *d*-wave Cooper pairing. For simplicity, we argue in terms of two-dimensional first Brillouin zone and Fermi surface (FS), neglecting the *c*-axis quasiparticle dispersion, which should be taken into account, in principle.[41] Since the superconducting *d*-wave, Δ, and the dielectric, Σ, order parameter have different momentum dependences, their mutual presence is no longer reduced to a combined gap $(\Sigma^2+\Delta_s^2)^{1/2}$ as for isotropic superconductivity.[21]

In the presented theory, superconductivity is described in the framework of weak-coupling model with the Hamiltonian given below. In accordance with angle-resolved photoemission spectroscopy (ARPES)[42–45] and STM (Refs. 15 and 46–52) data, the CDW (electron-hole) pairing is supposed to be restricted to momentum ranges near flat-band regions on the FS, antinodal from the viewpoint of the four-lobe *d*-wave gap function $\Delta(T)\cos 2\theta$.[53] In those regions, the degeneracy relations

$$\xi_1(\mathbf{p}) = -\xi_2(\mathbf{p}+\mathbf{Q}_i) \tag{1}$$

take place between pairs of mutually coupled quasiparticle branches, being a reason of dielectric gapping (see below). Relationship (1) is fulfilled, e.g., in the tight-binding approximation for a two-dimensional square lattice, corresponding to conducting CuO layers.[54–56] The wave vectors $\mathbf{Q}_i$ $(i=1,2)$ connect the congruent (nested) FS sections in pairs and Planck's constant $\hbar=1$.

The $4a_0\times 4a_0$ charge-ordered checkerboard state was discovered, for instance, in photoemission studies of $Ca_{2-x}Na_xCuO_2Cl_2$, a_0 being the lattice constant in the CuO_2 plane.[42] STM reveals a static charge modulation with the wave vectors $\mathbf{Q}=(\pm 2\pi/4.2a_0,0)$ and $(0,\pm 2\pi/4.2a_0)$—with an accuracy of 15%—in $Bi_2Sr_2CaCu_2O_{8+\delta}$ ($T_c\approx 89$ K for optimally doped samples)[15] and CDWs—with an incommensurate period and the CDW wave vectors $\mathbf{Q}$ depending on the oxygen doping degree—in $Bi_2Sr_{1.4}La_{0.6}CuO_{6+\delta}$ ($T_c^{\max}\approx 29$ K).[57] The same method disclosed non-dispersive (energy-independent) checkerboard CDWs in $Bi_{2-y}Pb_ySr_{2-z}La_zCuO_{6+x}$ ($T_c\approx 35$ K for the optimally doped composition).[12] In this case, $\mathbf{Q}$ substantially depends on doping, with its absolute value rising from $\pi a_0^{-1}/6.2$ in an optimally doped sample to $\pi a_0^{-1}/4.5$ for an underdoped sample with $T_c\approx 25$ K. It was easily explained by taking into account the shrinkage of the hole FS with a reduction in hole number[12] so that the vector $\mathbf{Q}$ that links the flat nested FS sections grows, which means a decrease in the

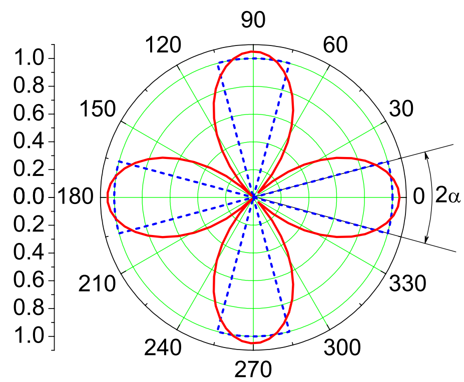

FIG. 1. (Color online) Order parameter maps for a bare d-wave superconductor (Δ, solid curve) and a partially gapped CDW metal (Σ, dashed curve).

CDW period. A combination of photoemission and STM techniques showed[9] how the pseudogap (the CDW gap, in our interpretation) above T_c in $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$ transforms into two gaps coexisting in real and momentum spaces below T_c. It should be noted that a competition between CDW and superconductivity in cuprates was supposed as early as in 1987 on the basis of heat capacity and optical studies.[1] The similarity in this respect between high-T_c oxides and dichalcogenides, where CDWs are ubiquitous,[58–64] was first noticed by Klemm.[11,65]

The flat FS sections themselves, which are inferred from photoemission experiments,[13] can be reproduced by tight-binding electron band calculations with “dressed” parameters, which take strong correlation effects into account.[66] It means that any model, which is based on the realistic two-dimensional FS with flat sections and concomitant nesting effects, implicitly makes allowance for many-body correlations between quasiparticles.

Thus, we arrive at a CDW checkerboard state (symmetric with respect to $\pi/2$ rotations) with four sectors in the momentum space centered at the superconducting lobes and with an opening 2α each ($\alpha < \pi/4$). It should be noted that the vectors $\mathbf{Q}_i$ depend on doping, which was explicitly shown for $Bi_2Sr_2CuO_{6+\delta}$.[52] The dielectric (CDW-induced) order parameter is $\Sigma(T)$ inside the 2α cones, being angle-independent here, and zero outside (see Fig. 1).

For the materials discussed above, the model mean-field Hamiltonian for a partially gapped CDW d-wave superconductor takes the form (see the derivation and discussion in Refs. 4, 29, 33–35, 67, and 68)

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{d\text{-BCS}} + \mathcal{H}_{\text{CDW}}, \quad (2)$$

$$\mathcal{H}_0 = \sum_{i=1,2,3} \sum_{\mathbf{p}\alpha} \xi_i(\mathbf{p}) a^\dagger_{i\mathbf{p}\alpha} a_{i\mathbf{p}\alpha}, \quad (3)$$

$$\mathcal{H}_{d\text{-BCS}} = -\sum_{i=1,2,3} \sum_{\mathbf{p}} \Delta(T) f(\mathbf{p}) (a^\dagger_{i\mathbf{p}\uparrow} a^\dagger_{i-\mathbf{p}\downarrow} + a_{i-\mathbf{p}\downarrow} a_{i\mathbf{p}\uparrow}, \quad (4)$$

$$\mathcal{H}_{\text{CDW}} = -\sum_{i=1,2} \sum_{\mathbf{p}\alpha} \Sigma(T) (a^\dagger_{i\mathbf{p}+\mathbf{Q}\alpha} a_{i\mathbf{p}\alpha} + a^\dagger_{i\mathbf{p}\alpha} a_{i\mathbf{p}+Q\alpha}). \quad (5)$$

Here $a^\dagger_{i\mathbf{p}\alpha}(a_{i\mathbf{p}\alpha})$ is the creation (annihilation) operator of a quasiparticle in the ith branch with the quasimomentum $\mathbf{p}$ and the spin projection $\alpha = \pm\frac{1}{2}$; the subscripts $i=1,2$ correspond to the nested FS sections, for which Eq. (1) is fulfilled, whereas index $i=3$ is applied to the rest of the FS with the conventional quasiparticle dispersion relation $\xi_3(\mathbf{p})$. The angular factor $f(\mathbf{p}) = \cos 2\theta$ describes the d-wave symmetry of the superconducting order parameter $\Delta(T)\cos 2\theta$, in accordance with the aforesaid.

The adopted phenomenological approach can describe several microscopic background models. For instance, in a one-dimensional metal, a Peierls-Fröhlich instability might occur when a dielectric gap is formed on the FS due to the formation of a CDW and a concomitant periodic lattice distortion.[69–73] Here, the electron-phonon interaction is a driving force, phonon softening being a formal reason of the phase transition. For a one-dimensional noninteracting electron gas, a static polarization operator $\Pi_0(\mathbf{q})$ has a logarithmic singularity at $q = 2k_F$, which inevitably causes the state instability.[74] Here, k_F is the Fermi wave number. This picture is only a crude approximation. Actually, if electron-electron interaction is taken into account, the situation becomes much more complicated because the resulting $\Pi(\mathbf{q})$ that governs bare phonon softening contains $\Pi_0(\mathbf{q})$ both in its numerator and denominator.[75] Whether $\Pi(\mathbf{q})$ diverges or not depends on a relationship between different matrix elements of Coulomb and electron-phonon interactions, thus imposing certain *criteria* on Peierls transition.[76–80] Any inevitable deviations from electronic one-dimensionality due to interchain Coulomb interaction or (and) electron hopping between neighboring chains lead to warping of the FS and reduction of its flat-section area.[81,82] Nevertheless, the Peierls instability preserves in other, more realistic models[83,84] and has been observed in plenty of materials.[85,86] Nesting conditions (1) are more difficult to be achieved in quasi-two-dimensional crystals although pronounced CDWs appear in such objects as well, resulting in insulating or less-metallic properties below a dielectric (structural) transition.[60,63,87–90] In our mean-field approach, we neglect order-parameter fluctuation effects, often being substantial both in quasi-one-dimensional and quasi-two-dimensional materials, where conventional “pseudogapping” emerges above mean-field critical temperatures.[59,62,63,81,83,86,91,92]

The scenario with the Peierls transition is not a unique one, leading to the same mean-field picture described by Hamiltonian (5). We mean the excitonic insulator model, where Coulomb attraction between electrons and holes result in a reconstructed state with a dielectric gap in the quasiparticle spectrum.[93–97] If the relevant hole maximum and electron minimum are located at different points of the Brillouin zone in the parent phase, the emerging gapping is accompanied by the appearance of CDWs or SDWs.[98,99] Earlier it was suggested that the semimetallic $1T-TiSe_2$ compound with CDWs below 200 K is an excitonic insulator,[100,101] and Cu_xTiSe_2 is a partially gapped excitonic superconductor.[102]

As for the *d*-wave term [Eq. (4)] in the Hamiltonian, it is a mean-field form of the phenomenological superconducting Hamiltonian in the standard weak-coupling approach.[103,104] The results presented below do not depend on a possible microscopic background, irrespective of its spin fluctuation (as the overwhelming majority of experts[105–112] think), electron phonon (according to the minority viewpoint[113–119]), electron plasmon,[120] or even more involved, combined[121] origin. Plenty of arguments (both pro and contra) concerning various mechanisms of superconductivity and even the very existence of an effective bosonic "glue" in high-T_c oxides were presented in Refs. 122–126, going, generally speaking, far beyond the scope of the problems studied here. In particular, it should be noted that strong on-site Coulomb (Hubbard) correlations are not included into our Hamiltonian (2), thus allowing a double lattice-site occupancy, which is not the case, e.g., for the $t-J$ model, where strong correlations prohibit spin-up and spin-down electrons from their location on the same site.[127–129] Phase diagrams for the latter model, which is quite different from ours, demonstrate, nevertheless, the coexistence of *d*-wave superconductivity with incommensurate or commensurate CDWs of various spatial symmetries.[130–133]

Let us assume that, except for the directions given by the vectors $\mathbf{Q}_1$ and $\mathbf{Q}_2$, both CDWs are identical and can be described by a single order parameter Σ. Then, the Dyson-Gor'kov equations for normal and superconducting Green's functions for a system with electron-hole (Peierls or excitonic) and *d*-wave Cooper pairings were solved in the same straightforward manner as in the *s*-wave case,[4,21] and the solutions turned out to be quite similar, with an accuracy to the replacement of $\Delta(T)$ by $\Delta(T)\cos 2\theta$. The standard self-consistency equations for $\Delta(T)$ and $\Sigma(T)$ are coupled and take the form

$$\int_0^{\mu\pi/4} I_M(\sqrt{\Sigma^2+\Delta^2\cos^2 2\theta},T,\Sigma_0)d\theta=0, \tag{6}$$

$$\int_0^{\mu\pi/4} I_M(\sqrt{\Sigma^2+\Delta^2\cos^2 2\theta},T,\Delta_0)\cos^2 2\theta d\theta + \int_{\mu\pi/4}^{\pi/4} I_M(\Delta\cos 2\theta,T,\Delta_0)\cos^2 2\theta d\theta=0, \tag{7}$$

where the Boltzmann constant $k_B=1$, $\mu=4\alpha/\pi$ is the apparent (see below) dielectrically gapped portion of the FS ($0<\mu<1$), Δ_0 and Σ_0 are the bare superconducting and CDW order parameters, respectively (each of them determines the corresponding gapping in the absence of the competing phenomenon),

$$I_M(\Delta,T,\Delta_0)=\int_0^\infty\left(\frac{1}{\sqrt{\xi^2+\Delta^2}}\tanh\frac{\sqrt{\xi^2+\Delta^2}}{2T}-\frac{1}{\sqrt{\xi^2+\Delta_0^2}}\right)d\xi \tag{8}$$

is the Mühlschlegel integral, the root of which $\Delta=s\text{Mü}(\Delta_0,T)$ is the well-known gap dependence of a BCS superconductor. Note that, according to the adopted model of partial dielectric gapping, $\Delta(T)$ is suggested to exist over the whole FS [summation in Eq. (4) is carried out over sections $i=1,2,3$], whereas $\Sigma(T)$ exists only at its degenerate nested sections [summation in Eq. (5) is carried out over sections $i=1,2$] inside the 2α cones defined above. In the absence of superconductivity—e.g., above T_c but below the CDW transition temperature, T_{CDW0} (in cuprates, pseudogaps generally emerge in the normal state[134,135])—Eq. (6) ceases to depend on μ, and $\Sigma(T)$ equals the BCS-like function $s\text{Mü}(\Sigma_0,T)$ with $\Sigma_0=\frac{\pi}{\gamma}T_{CDW0}=2W\exp[-1/V_{CDW}\mu N(0)]$, where $\gamma=1.78\ldots$ is the Euler constant, V_{CDW} is the matrix element of the electron-hole pairing interaction, W is the cutoff energy in the CDW channel, and $N(0)$ is the total normal-state electron DOS. The analysis of the generic $T-\delta$ phase diagram shows that both Σ_0 and μ reduce with doping, whereas the holelike FS pockets centered at the $(\pi/a_0,\pi/a_0)$ point of the Brillouin zone shrink for every specific high-T_c oxide (see, e.g., Ref. 52). Thus, the doping dependences of observed quantities are mainly governed by the corresponding variations of the control parameter μ. We emphasize that our model qualitatively correctly describes all oxide compositions with nonzero Fermi arcs between 2α cones, leaving, e.g., localization phenomena[136] beyond the scope of consideration.

On the other hand, in the absence of CDW gapping, Eq. (7) becomes a *d*-wave gap equation,

$$\int_0^{\pi/4} I_M(\Delta\cos 2\theta,T,\Delta_0)\cos^2 2\theta d\theta=0, \tag{9}$$

the solution of which $\Delta=d\text{Mü}(\Delta_0,T)$ is also known.[103,104] In particular, the critical temperature is $T_{c0}=\frac{2\Omega\gamma}{\pi}\exp[-1/V_{BCS}N(0)]$, where Ω and V_{BCS} are the Cooper-pairing cutoff and interaction amplitude, respectively. From Eq. (9), it follows that, in agreement with Ref. 103, $(\Delta_0/T_{c0})_d=(2/\sqrt{e})(\pi/\gamma)$, revealing a modified "*d*-wave" BCS-ratio different from the *s*-pairing value

$$\left(\frac{\Delta_0}{T_{c0}}\right)_s=\frac{\pi}{\gamma}\approx 0.824\left(\frac{\Delta_0}{T_{c0}}\right)_d. \tag{10}$$

Here e is the base of natural logarithm. It is evident that our model takes into account many-body correlations both explicitly (the emergence of two pairings caused by electron-phonon and Coulomb interaction between charge carriers) and implicitly (via the renormalization of the parameter μ). The renormalized nature of μ, which is the phenomenological parameter of our model, means that its values are "dressed" by correlation effects[66] rather than are given by a band theory alone, e.g., in the tight-binding approximation, and they can be directly inferred from the experiment (see below).

III. RESULTS

Due to the different order-parameter symmetry, readily seen from Eqs. (6) and (7), the situation is mathematically more involved than that for CDW *s*-wave superconductors, where a simple relationship $\Delta_s^2(T)+\Sigma_s^2(T)=[s\text{Mü}(\Sigma_0,T)]^2$ takes place between their order parameters.[21] *Prima facie* subtle numerical differences between $d\text{Mü}(\Delta_0,T)$ and

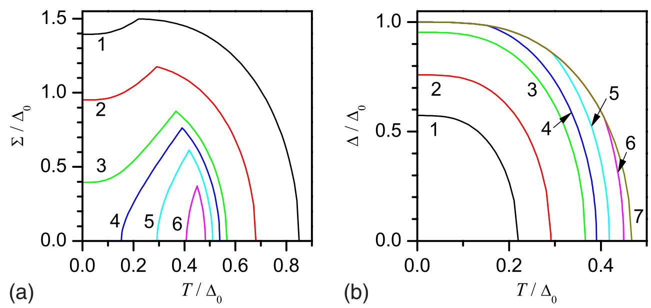

FIG. 2. (Color online) Temperature, T, dependences of the normalized (a) charge-density wave (CDW) Σ and (b) superconducting Δ gap functions. The values of Σ_0/Δ_0, where $\Sigma_0=\Sigma(T=0)$ in the absence of superconductivity and $\Delta_0=\Delta(T=0)$ in the absence of CDW, are 1.5 (1), 1.2 (2), 1 (3), 0.95 (4), 0.9 (5), 0.85 (6), and 0.8 (7); the portion of the Fermi surface gapped by CDWs $\mu=0.3$.

sMü(Σ_0,T) lead to conspicuous physical consequences. Indeed, the dependences $\Delta(T)$ and $\Sigma(T)$ found from Eqs. (6) and (7) and shown in Fig. 2 differ *qualitatively* from their counterparts $\Delta_s(T)$ and $\Sigma_s(T)$ in a certain range of model parameters. (In this paper, for brevity, we do not introduce a natural subscript "d" for quantities with d-wave pairing origin.) Panel (a) demonstrates that a reduction of the bare parameter Σ_0, keeping Δ_0 and μ constant, results in the transformation of $\Sigma(T)$ with a cusp at $T=T_c$ and a concave region at $T<T_c$ (as it takes place for CDW s-superconductors in the whole allowable parameter range[21]) into curves describing a peculiar re-entrant CDW state. The re-entrance was found in the framework of the simplest possible model including two competing order parameters with different spatial symmetry. At the same time, the CDW structures in real systems may be much more complicated with non-monotonic T dependences even in the absence of superconductivity.[13]

Let us formulate conditions necessary to observe the crossover between the conventional and the reentrant $\Sigma(T)$ behavior. First, relationship (10) means that $\Delta(T)/\Delta_0$ for conventional d superconductors is steeper than $[\Delta(T)/\Delta_0]_s$. In our case, it means that $\Delta(T)/\Delta_0$, when the CDW disappears, is steeper than $\Sigma(T)/\Sigma_0$ in the absence of superconductivity. Hence, for the CDW phase to exist (the upper critical temperature $T^u_{\mathrm{CDW}}>0$), it should be $T^u_{\mathrm{CDW}}=\frac{\gamma}{\pi}\Sigma_0>T_{c0}=\frac{\sqrt{e}\gamma}{2\pi}\Delta_0$. As a consequence, the following constraint on the model parameters should be fulfilled: $\Sigma_0>\frac{\sqrt{e}}{2}\Delta_0$. The coexistence superconductivity–CDW was not involved in these reasonings, so the inequality does not include the control parameter μ. Obviously, T^u_{CDW} thus defined coincides with $T_{\mathrm{CDW}0}$.

Second, below the lower critical temperature of the CDW reentrance region, T^l_{CDW}, if any, Eq. (7) defines $\Delta(T)=d$Mü(Δ_0,T), and we should use Eq. (6) with $T=T^l_{\mathrm{CDW}}$ and $\Delta(T^l_{\mathrm{CDW}})=d$Mü$(\Delta_0,T^l_{\mathrm{CDW}})$ to determine $T^l_{\mathrm{CDW}}(\Delta_0,\Sigma_0,\mu)$ numerically. The crossover value of Σ_0^{cr} when $T^l_{\mathrm{CDW}}=0$ corresponds to the separatrix dividing possible $\Sigma(T)$ curves [see Fig. 2(a)] into two types: reentrant and non-re-entrant. Equation (6) brings about $\Sigma_0^{\mathrm{cr}}=\Delta_0\exp[\frac{4}{\mu\pi}\int_0^{\mu\pi/4}\ln(\cos 2\theta)d\theta]$. For the curves in Fig. 2, $\mu=0.3$ was chosen so that we obtain the re-entrance range $0.824\Delta_0<\Sigma_0<0.963\Delta_0$, which agrees with numerical solutions. We emphasize that CDWs survives the competition with d-wave superconductivity even at $\Sigma_0/\Delta_0<1$, which is not the case for stronger isotropic Cooper pairing.[21]

In Fig. 2(b), the concomitant $\Delta(T)$ dependences are depicted. One sees how d-wave superconductivity, suppressed at large Σ_0s, recovers in the re-entrance parameter region. Therefore, two regimes of CDW manifestation can be observed in superconductors. In both cases, the CDW is seen as a pseudogap above T_c in photoemission and tunnel experiments.[5,6] But the corresponding dip-hump structure at low T may either be observed or not, depending on whether the reentrance occurs. Hence, samples exhibiting clear-cut pseudogaps above T_c and no traces of dip-hump structures at low T should be tested for Σ-re-entrance phenomena in the vicinity of T_c, e.g., by applying external magnetic fields.

It is important to perceive that the reentrance is a consequence of different symmetries inherent to CDW and superconducting order parameters in our case. Indeed, different symmetries result in different T dependences of bare Δ and Σ, being the dMü(Δ_0,T) and sMü(Σ_0,T) functions, respectively. Hence, when lowering T, a more abrupt growth of $\Delta(T)$—in comparison with that of $\Sigma(T)$—in the coexistence region of the phase diagram, together with the destructive interference between the phenomena concerned, totally suppresses CDWs. If both order parameters are assumed to include an identical factor in the momentum space, e.g., $f(\mathbf{p})=\cos 2\theta$, the effect does not exist.[137,138]

In this connection, we note also that the very approach of Refs. 137 and 138 differs from ours in many respects. First, the corresponding authors select both order parameters as d-wave ones, which, as has been indicated, rules out the reentrant behavior of $\Sigma(T)$. The conventional (non-reentrant) character of coupled dependences $\Delta(T)$ and $\Sigma(T)$ was demonstrated earlier for various exotic angular factors in application to a number of heavy fermion compounds.[139] Second, not only the form of order-parameter angular dependences in Refs. 137 and 138 but also their orientation relative to coordinate axes in the two-dimensional momentum space was assumed identical. At the same time, the d-wave electron-hole gapping is *complete* by definition rather than partial, the latter being appropriate to our model; namely, it is uniform in each of four equivalent cones in the two-dimensional Brillouin zone. (The influence of point nodes is negligible as regards the issue of gapping completeness.) These circumstances give almost no chances for superconductivity to survive against the CDW background, as was shown earlier for the case of isotropic order parameters (complete dielectric and superconducting gappings)[95,96,140,141] and is also true for d-wave order parameters with identical angular functions.[137]

Therefore, an additional shift $\delta\mu$ of the chemical potential was incorporated into kinetic term [Eq. (3)] of the Hamiltonian and interpreted as a term describing doping.[137,138] Although the inclusion of doping into consideration is possible, in principle, the $\mathbf{k}$ independence of the energy offset $\delta\mu$ signifies that there is a nonvanishing quasiparticle DOS at $E=0$ on the whole FS in a normal metal or a superconductor, gapped by a d-like charge-density wave.[138] It means that a CDW gap appears on the new FS (in the doped state), whereas quasiparticles from degenerate sections of the

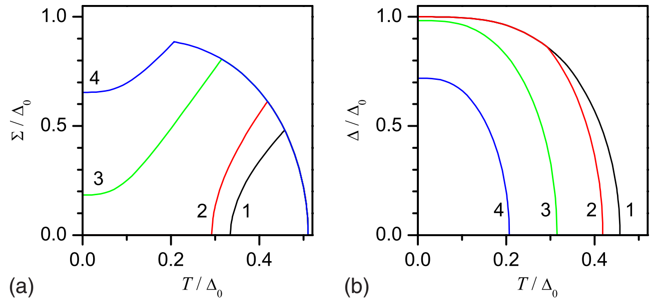

FIG. 3. (Color online) The same as in Fig. 2 but for $\Sigma_0/\Delta_0=0.9$ and $\mu=0.1$ (1), 0.3 (2), 0.5 (3), 0.6 (4).

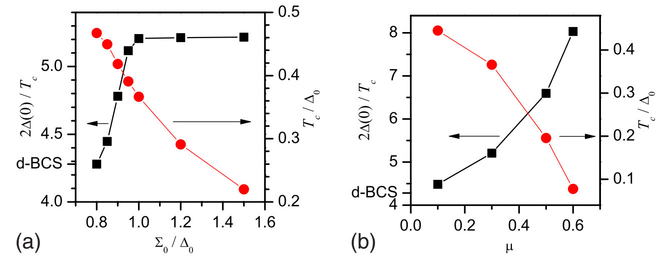

FIG. 4. (Color online) Dependences of $2\Delta(0)/T_c$ (squares) and T_c/Δ_0 (circles) on Σ_0/Δ_0 (panel a, $\mu=0.3$) and μ (panel b, $\Sigma_0/\Delta_0=1$). T_c is the superconducting critical temperature, $d-\text{BCS}\approx 4.28$ is a "pure" d-value.

parent FS remain responsible for the CDW instability, whatever strong is the doping. That scenario does not seem reasonable. Moreover, our viewpoint, according to which the Fermi level lies inside the dielectric gap, and non-degenerate FS sections (outside the cones) remain intact below the CDW critical temperature T_d in order for the Cooper instability to develop at $T_c<T_d$, is directly (ARPES) or indirectly confirmed by experiment.[142–148] It should also be noted that if the assumption of the overall chemical-potential shift[137,138] had been realized, the increase in electron DOS would have led to the CDW-induced increase in T_c.[80,149,150] But if so, such an increase could be detected by varying the doping level for the CDW effects to change substantially. In actual fact, the pseudogap growth anticorrelates with T_c,[4] being one more argument against the complete CDW-gapping scenario.

Turning back to our approach, we want to emphasize that in order to change over between different regimes (re-entrant—non-re-entrant) in cuprates, one can use either hydrostatic pressure or doping. In both cases, μ is the main varying parameter (our μ has nothing to do with $\delta\mu$ in Refs. 137 and 138). In Fig. 3, the curves $\Sigma(T)$ and $\Delta(T)$ are shown for $\Sigma_0/\Delta_0=0.9$ and various μs. It is readily seen how drastic is the low-T depression of Σ by superconductivity when the dielectrically gapped FS sectors are small enough. Doping $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Ref. 144) and $(Bi,Pb)_2(Sr,La)_2CuO_{6+\delta}$ (Ref. 147) with oxygen was shown to sharply shrink the parameter μ. Specifically, the values of the parameter μ calculated on the basis of pseudogap momentum dependences are as follows: (i) $\mu=0.82$, 0.67, and 0.51 for UD75 K, UD92 K, and OD86 K samples of $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BSCCO), respectively; (ii) $\mu=0.67$, 0.56, and 0.33 for UD23 K, optimally doped 35 K, and OD29 K samples of $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_8$, respectively. Here, samples are denoted by their superconducting critical temperature, whereas UD and OD mean "underdoped" and "overdoped" samples, respectively. Unfortunately, we could not infer the charge carrier concentration, n_c, from the data reported in Refs. 144 and 147. Note that quite unexpectedly the magnitudes of dielectrically gapped FS portions turned out larger for oxides with higher T_cs.

There is also a viewpoint that pseudogaps do not exist in optimally and overdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ samples.[151] Note that the $\Delta(T)$ dependences are distorted by CDWs, and they do *not* coincide with the scaled parent curve—dMü(T), in this case—in contrast to what is observed for CDW s superconductors.[21] Therefore, various observed forms of $\Delta(T)$ per se cannot testify unambiguously to the superconducting pairing symmetry. Moreover, the problem of a genuine order-parameter symmetry in cuprates is far from being solved[152–164] so that their superconductivity might be, e.g., a mixture of s- and d-wave contributions.[165,166]

It is evident that different strengths of CDW-imposed suppression of the superconducting energy gap in the electron spectrum Δ and the critical temperature T_c must change the ratio $\Delta(0)/T_c$—the benchmark of *weak-coupling* superconductivity [see Eq. (10)]. If one recalls that this ratio in CDW s superconductors remains the same as in pure s ones,[21] the situation becomes very intriguing. In Fig. 4(a), the dependences of $2\Delta(0)/T_c$ and T_c/Δ_0 ratios on Σ_0/Δ_0 are shown. One sees that $2\Delta(0)/T_c$ sharply increases with Σ_0/Δ_0 for $\Sigma_0/\Delta_0\leq 1$ and swiftly saturates for larger Σ_0/Δ_0, whereas T_c/Δ_0 decreases almost evenly. The saturation value proves to be 5.2 for $\mu=0.3$. We stress that such large enhancement of $2\Delta(0)/T_c$ agrees well with experimental data[23–28] for cuprates and *cannot* be achieved taking into account strong-coupling electron-boson interaction effects for reasonable relationships between T_c and certain effective boson frequencies ω_E (Ref. 167).

To be more specific, the quantity $[2\Delta(0)/T_c]_{s,\text{sc}}$ (the abbreviation "sc" stands for "strong coupling") for isotropic superconductors, renormalized by conventional (electron-phonon) strong-coupling corrections, can be approximated as follows:[168]

$$\left(\frac{2\Delta(0)}{T_c}\right)_{s,\text{sc}} = 3.53\left[1+12.5\left(\frac{T_c}{\omega_E}\right)^2\ln\left(\frac{\omega_E}{2T_c}\right)\right], \qquad (11)$$

where ω_E is the logarithmic average energy of phonons in this case. For d-wave superconductors, there are corrections of the same kind as Eq. (11) so that the corresponding numerical calculations lead to the saturation of $(2\Delta(0)/T_c)_{d,\text{sc}}$ at about 6.5 for $T_c/\omega_E\approx 0.3$, whereas, for larger T_c/ω_E, the ratio $(2\Delta(0)/T_c)_{d,\text{sc}}$ starts to drop.[169] Hence, in the framework of the strong-coupling d-wave theory, one cannot describe observed values of $2\Delta(0)/T_c$ larger than 6.5. We can also hardly accept the existence of a saturating value $T_c/\omega_E\approx 0.3$ (Ref. 169) because it is practically meaningless.

Another scenario, which is based on the destruction of the alternating-sign superconducting order parameter by impurity scattering approximated by collective boson modes[170] could not also explain high values of $2\Delta(0)/T_c$, inherent, e.g., to underdoped BSCCO.[24,25] At the same time, our weak-coupling model is *sufficient* to reproduce large $2\Delta(0)/T_c$ values for cuprates, with possible strong-coupling effects[167–169] resulting in smaller corrections as compared to a huge effect of CDWs.

A singular energy dependence of the normal-state electron DOS near the FS, for instance, near the Van Hove anomalies in low-dimensional electron subsystems,[171] might have been another possible alternative reason of high $2\Delta(0)/T_c$ ratios. It turned out, however, that, at least in the weak-coupling (BCS) approximation for *s*-wave Cooper pairing, the ratio $2\Delta(0)/T_c$ is not noticeably altered.[172,173] Moreover, calculations in the framework of the strong-coupling Eliashberg theory[174] showed that the Van Hove singularity influence on T_c is even smaller than in the BCS limit.[175] On the other hand, weak-coupling calculations for orthorhombically distorted hole-doped cuprate superconductors (without CDWs) demonstrated that $2\Delta(0)/T_c$ can be estimated as an intermediate between the *s*- and *d*-wave limits,[176] being smaller than needed to explain the experiment. It means that our approach remains so far the only one capable of explaining high ($2\Delta(0)/T_c \approx 5 \div 8$) and even larger values for cuprates.[28]

It is instructive from the methodological point of view to mention a previous unsuccessful attempt to explain the increase in $2\Delta(0)/T_c$ by the pseudogap influence.[177] The authors of Ref. 177, similarly to what was suggested in Refs. 137–139 discussed above, assumed the *identical* *d*-wave symmetry of both superconducting $\Delta(T)$ and pseudogap order parameters. The former authors considered, however, that pseudogaps were *T*-independent constants E_g. This assumption led to the lack of self-consistency and to an unnecessary restriction imposed on E_g, namely, $E_g \lesssim 0.53\Delta_0(T=0)$, where $\Delta_0(T=0)$ is the parent superconducting order-parameter amplitude. At the same time, it is well known that, for existing CDW superconductors, the strength of the CDW instability is at least not weaker than that of its Cooper-pairing counterpart.[4] We should emphasize once more that the *main peculiarity* of our model, dictated by the observations, which led to the adequate description of thermodynamic properties for *d*-wave superconductors with CDWs, is the distinction between the relevant order-parameter symmetries.

The μ dependences of $2\Delta(0)/T_c$ and T_c/Δ_0 are shown in Fig. 4(b). They illustrate that the ratio $2\Delta(0)/T_c$ can reach rather large values if the dielectric gapping sector is wide enough. This growth is however limited by a drastic drop of T_c leading to a quick disappearance of superconductivity. We think that it is exactly the case of underdoped cuprates when a decrease in T_c is accompanied by a conspicuous widening of the superconducting gap. For instance, such a scenario was clearly observed in break-junction experiments for $Bi_2Sr_2CaCu_2O_{8+\delta}$ samples with a large doping range.[178]

As has already been indicated above, various photoemission and tunneling measurements for different cuprate families[23–28] demonstrate unconventionally high ratios $2\Delta(0)/T_c \approx 5-8$ with a typical average value $2\Delta(0)/T_c \approx 5.5$.[26] From our Fig. 4(b), one can see that the latter value corresponds to $\mu \approx 0.35$ at $\Sigma_0/\Delta_0 = 1$. The other curve readily gives $T_c/\Delta_0 \approx 0.35$. Since $\Delta_0/T_{c0} \approx 2.14$ for a *d*-wave superconductor (see above), we obtain $T_c/T_{c0} \approx 0.75$, being quite a reasonable estimation of T_c reduction by CDWs.

IV. CONCLUSIONS

To summarize, we have shown that *d*-wave superconductivity emerging on the whole FS can coexist with CDW gapping restricted to the antinodal FS regions. The interplay between Δ and Σ differs from that for CDW *s*-wave superconductors. Namely, the dependence $\Sigma(T)$ may become reentrant in certain ranges of the problem parameters, vanishing at low *T*. On the other hand, the actual $\Delta(T)$ is also substantially distorted by detrimental CDW influence so that the ratios $2\Delta(T=0)/T_c$ become anomalously large, substantially exceeding the BCS value for *d*-wave superconductors. This constitutes a long-expected explanation for experimental values $2\Delta(0)/T_c \approx 5-8$ typical of high-T_c oxides.[23–28] Our conclusions can be verified by measuring the dependences of the ratio $2\Delta(0)/T_c$ on the doping level for selected cuprates and provided that the superconducting and CDW gaps (pseudogaps) are not confused. It seems instructive to find correlations between doping dependences of T_c, μ, and $2\Delta(0)/T_c$. In this connection, we emphasize once more that the input parameter μ changes gradually with the nonstoichiometry parameter δ or, equivalently, the charge carrier concentration n_c. Moreover, the dependences $\mu(\delta)$ and $\mu(n_c)$ can be relatively easy inferred from the Fermi surface reconstructed, e.g., from ARPES measurements.[144,147] Therefore, our theory can be verified by checking the correspondence between the observed peculiarities predicted here and their description as the solutions of the system of Eqs. (6) and (7).

It is worthwhile noting that, within certain doping ranges, superconducting cuprates demonstrate[179] the *coexistence* of Cooper pairing with SDWs rather than CDWs. Our self-consistent approach can be generalized to study that situation as well.

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